

The Effect of Probabilistic Thinking on the Ability of Undergraduate Students of Mathematics Education in Solving Binomial Distribution Problems

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Abstract

Mathematical statistics course I is one of the compulsory subjects taught in tertiary institutions, one of the sub-matters is distribution which is divided into 2, namely discrete distribution and continuous distribution. The discrete distribution is one of the materials that is poorly understood, especially regarding the binomial distribution, so that students' mastery of the material becomes very lacking. Efforts to overcome this problem include maximizing probabilistic students' thinking and solving problems on the binomial distribution so that undergraduate students can understand the concepts taught by the supervisor of the course. The purpose of this research is to find out how much probabilistic students' thinking skills are in solving binomial distribution problems. While solving the problem of the binomial distribution is an ability in the problem solving process by using all knowledge in solving the binomial distribution problem through 4 step indicators, namely random experiments, sample space, events, and the probability of an event and skills that already exist and synthesize them so that the goal of solving is achieved. the problem with the binomial distribution is the chance of getting success or failing. The method used in this research is a quantitative approach. The type of research used is a case study. The population and sample in this study were fourth semester undergraduate students, Mathematics Education Study Program, FKIP USN Kolaka Class of 2020/2021, a total of 24 students. The instrument used for data collection is a probabilistic thinking test and a probability distribution problem solving test. The results of this study obtained the value of $F_{count} = 7.662$ with a significance of $0.011 < \alpha = 0.05$ then H_0 was rejected and H_a was accepted. This means that students' probabilistic thinking has a significant influence in overcoming the problem of the binomial distribution. While the correlation value (r) of 0.508 is included in the sufficient criteria. The coefficient of determination (r^2) = 0.258 or 25.80%, meaning that there is an influence between the independent variable and the dependent variable and the remaining 74.20% is determined by other factors. The regression equation for variable Y on variable X is: $\hat{Y} = 48,020 + 0,437X$. A constant of 48,020 states that if the value of critical thinking is 0,437 then the student's ability to solve the binomial distribution problem is 48,020. The regression coefficient of 0.437 states that each additional value of 1 in critical thinking will increase undergraduate students' ability to solve the problem of the binomial distribution of 0.437.

Keywords: Probabilistic Thinking, Undergraduate Students of Mathematics Education, Problem Solving, Binomial Distribution

INTRODUCTION

More formally, thinking is cognitive rearrangement or manipulation both information from the environment and symbols that are stored in the long term memory (Solso et al., 1995). Thinking is a process in which new mental representations formed through the transformation of information with complex interactions with mental attributes such as judgment, abstraction, logic, imagination, and problem solving (Solso et al., 1995). This understanding shows that there are three basic views about thinking, namely (1) thinking is cognitive, that is, it arises

internally in the mind but can predicted from behavior, (2) thinking is a process that involves some manipulation of knowledge in cognitive systems, and (3) thinking is directed and produces behavior that solves problems or is directed at solutions.

Included in this is the process of thinking mathematically. The term mathematical thinking is defined as a way of thinking with regard to the process of mathematics (doing math) or a way of thinking in solving problems. mathematical tasks both simple and complex (Isoda & Katagiri, 2012) . Mathematical thinking is just one aspect of thinking in general. The mechanism of the process of thinking mathematically is the same as the process of cognition in general, namely translation, integration, planning and implementation. One of the mathematical thinking processes that is quite interesting to study is the probabilistic thinking process, especially the thinking processes of students who are categorized at the level of early adult thinking where the cognitive development of early adults or young adults is at the stage of postformal reasoning or post-formal reasoning which is characterized by thoughts that are dialectical. Dialectical ability, namely the ability to understand, analyze and find meeting points of ideas, ideas. theories, opinions and thoughts that are contradictory in nature so that individuals will be able to synthesize them into new and creative thoughts. The term probabilistic thinking is used to describe student thinking in response to various probabilistic problems. The word probability/chance refers to the level of one's belief in something that will happen. However, the belief referred to in opportunity is not a belief in the form of judgment, for example a belief about the "right/wrong" of someone's words, but rather a belief about the possibility of an outcome from a conceptual experiment.

Talking about opportunities, we are faced with an uncertain condition, but we are only given a hint or an illustration of how much we believe that an event can occur. The greater the probability value resulting from a calculation, the greater our confidence that the event will occur. Today, the prediction of the occurrence of a natural phenomenon is not something simple, but has gone through a very complex calculation process. Symptoms of an event are not only studied from one side, for example the influence of time, but also involve many variables related to the event. Therefore opportunities that are based on a scientific background can provide a higher level of confidence for people who need it.

In order to express an uncertainty or certainty, a mathematical model is needed which is theoretically expressed by a distribution or distribution. The probability value of an event in an experiment is spread between 0 and 1 or between 0% and 100%. If the probability/probability of an event A occurring is denoted by the notation $P(A)$ then, the probability [not A] or the complement of A, or the probability that an event A will not occur, is $1-P(A)$ (Goldstein & Rothschild, 2014; Viti et al., 2015) . Opportunity distribution is very useful for analyzing the occurrence of an event or event, if the event is finite then the distribution object is different from the event that is infinite (Khosravi et al., 2013) . The object of the probability distribution is a random variable where this object is a function that associates a real number to each element in the sample space (Effandi Zakaria & Normah Yusoff, 2009) . There are various kinds of opportunity distributions including discrete distributions and continuous distributions. A discrete distribution is a distribution in which a sample space has an infinite number of sample points, while a continuous distribution is a distribution in which a sample space has an infinite number of sample points (Vardeman et al., 1986) .

The binomial distribution is the Binomial distribution is a probability distribution that can be used when a sampling process can be assumed to be in accordance with the Bernoulli process (Walpole et al., 2011) . The binomial distribution comes from the binomial experiment, which is a Bernoulli process that is repeated n times and is independent of each other. The binomial distribution is a distribution of discrete random variables. Directly, a binomial

experiment has the following characteristics: 1) The experiment consists of n repeated attempts; 2) Every effort gives results that can be determined by success or failure; 3) The probability of success is expressed by θ , does not change from one attempt to the next; 4) Each business is independent of other efforts (Walpole et al., 2011) .

(Walpole et al., 2011) in his book explains that in the material probability, then presented sub material based on *Sample space*, *Event* and *Probability of an Event*. This is in line with some expert opinions that conclude that determining the probability of an event starts with a random experiment, determining the sample space to calculating the probability of an event (Nilsson et al., 2006); (Chernoff, 2009); (Khemlani et al., 2012); (Batanero, 2015) ; Lavenant & Santambrogio, 2019) ; (Teigen & Cool, 2020) , besides (Konold, 2002) consider randomness as a label used to associate many concepts such as experiment, sample space, event, to the probability of an event. In this sense, the word randomness refers to a collection of mathematical concepts and procedures, which can be applied based on a particular situation, while Nunes et al., (2014) in his research, states that four ideas of probability are the key to successful study in probability: 1) understanding randomness and its consequences; 2) analyze the sample space; 3) calculate probabilities as ratios; 4) develop correlational reasoning involving the coordination of the three previous ideas. Based on a number of theory above, then could constructed indicators of probabilistic thinking as measuring tool for something to be achieved in research. Indicator probabilistic thinking includes: Random experiment, Sample Space, Event Probability of an Event.

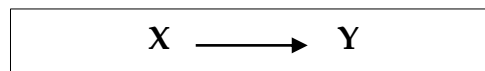
In the learning process, the binomial distribution material in probability courses is still relatively difficult to solve, this is in line with the results of initial interviews with the lecturers of the Mathematics Statistics 1 course that most students have difficulty determining the right steps to solve non-routine questions related to probability with respect to the matter of calculating the probability of an event using probability distributions such as random probability distribution and continuous probability distribution. This is in line with several previous studies which concluded that students had difficulty or constraint moment solve the problem probability (Beitzel & Staley, 2015; Galavotti, 2015; Lee & Yun, 2018; Usry et al., 2016) . besides that, performance student still not enough satisfying even though a student the has learn the basics probability moment in level medium (Danışman & Tanişli, 2017; Effandi Zakaria & Normah Yusoff, 2009; Yusuf et al., 2021) . The inaccuracy of students in working on and ignoring the basic concepts in solving these problems makes probability material difficult material, not only at the elementary level but up to the tertiary level, this is in line with the results of research by (Khemlani et al., 2012; Moreno & Cardeñoso , 2014) who stated that the teaching of probability theory lacks insight into the personal meaning of future mathematics teacher education related to probability and statistics because it does not provide opportunities for mathematics education students to find different perspectives on the nature and usefulness of this theory.

In solving problems, this formal operational thinker should be more systematic in developing hypotheses about why things happen the way they do, then test these hypotheses in a deductive way so that these things can be obtained in probabilistic thinking. Probabilistic thinking usually solves non-routine mathematical problems that contain uncertainty. There are three categories that show the ability of Higher Order Thinking (HOTS), namely: (1) raises the transfer of one concept to other concepts, (2) examine ideas and information critically, (3) use information to solve problems. Based on the HOTS category, probabilistic problem solving contains all three of these categories, this shows that solving probabilistic problems require higher order thinking skills or Higher Order Thinking Skills (HOTS)

Therefore, a way is needed to create learning situations that make students able to solve mathematical problems well, namely by using the Steps of Polya (1945) to be able to stimulate and train the probabilistic thinking skills of prospective teacher students in mathematics lectures, it is necessary to use methods or appropriate techniques in lectures so that students of Mathematics Education S1 as prospective teachers can use all their thinking potential. Problem solving is the right way in lectures to train students to think. Pehkonen, et al, (2008) , states that "Problem solving has been generally accepted as means for advancing thinking skills" Problem solving has been generally accepted as a way to improve thinking skills. In addition, NCTM (2000) states that "problem solving plays an important role in mathematics and should have a prominent role in the mathematics education ." Problem solving plays an important role in mathematics and should have a major role in mathematics education. Based on the problems faced by students in understanding the binomial distribution material, the researcher is interested in conducting research on the effect of probabilistic thinking on the ability of undergraduate mathematics education students in solving binomial distribution problems. It is hoped that the results of this study can be used as a guideline lecturers in improving aspects of probabilistic thinking so that in solving the problem of the binomial distribution it can be optimally achieved

RESEARCH METHODS

The method used in this research is a quantitative method. The aim is to see how much influence probabilistic thinking has on students' abilities in solving binomial distribution problems in the Mathematical Statistics I Semester IV FKIP USN Kolaka course. While the type of research that researchers do is the type of *Ex Post Facto* research. Which aims to determine students' probabilistic thinking skills in solving binomial distribution problems. This study uses a case study design, where this research will measure how much influence X (probabilistic thinking ability) has on and Y (solving the binomial distribution problem).



The population in this study were fourth semester students of class 2020/2021 Mathematics Education Study Program USN Kolaka, which consisted of 1 class with a total of 18 students. The instrument used in this research is a probabilistic thinking test and probability problem solving which consists of 4 items. Before the research was carried out, the validity and reliability of the instrument were first tested using Product Moment correlation and Cronbach's Alpha coefficient with the help of the SPSS Version 20.0 computer program. Tests were conducted on 24 undergraduate students to find out and measure probabilistic thinking skills and undergraduate students' abilities in solving binomial distribution problems.

The data analysis technique in this study used hypothesis testing with a simple linear regression test. Before using the hypothesis test, prerequisite tests are first carried out such as the normality test and linearity test as follows.

1. Normality Test

The normality test aims to test whether the sample used has a normal distribution or not. In a linear regression model, this assumption is indicated by the error value (e) which is normally distributed. A good regression model is a regression model that has a normal or close to normal distribution, so it is feasible to carry out statistical tests. The data normality test uses the Kolmogorov-Smirnov Test of Normality in the SPSS program. According to Santoso

(2012) the basis for decision making can be based on probability (Asymtotic Significance), namely:

- a. If the probability > 0.05 then the distribution of the regression model is normal.
- b. If the probability < 0.05 then the distribution of the regression model is not normal.

2. Linearity Test

The linearity test serves to find out whether between variables are independent and the dependent is linear or not. According to Santoso (2012) if the relationship is not linear, then the regression model will be biased when make predictions on the dependent variable. In SPSS this test is called Test for linearity with a significance level of 0.05. There are two ways of making decisions in the linearity test in the form of:

- a. The two variables are said to be linear if the significance is more than 0.05 so that the decision making is in the form of a linear relationship. Meanwhile, if the significance is below 0.05 then the decision making is in the form of no linear relationship between the two variables.
- b. Looking at the calculated F values and F tables, if the calculated F is greater than the F table then the conclusion is that there is no linear relationship between the two variables whereas if the calculated F is smaller than the F table then the conclusion is that there is a linear relationship between the two variables (Santoso, 2012).

3. Hypothesis Test

In this study there is one independent variable that will be tested to determine its effect on the dependent variable, so the regression analysis process is carried out using simple regression analysis. According to Sugiyono (2012) defines that: "*Simple regression is based on a functional or causal relationship of one independent variable with one dependent variable*" According to Sugiyono (2012) the simple linear regression equation set is as follows:

$$\hat{Y} = \alpha + \beta X$$

Information:

\hat{Y} = Solving the Binomial Distribution Problem

α = Constant

β = Regression coefficient

X = Variable probabilistic thinking of Mathematics Education undergraduate students

RESULT AND DISCUSSION

To find out how much the ability to think critically, the author uses the instrument The test is in the form of description questions which consist of 4 questions. The questions were distributed to 24 semester IV students, Batch 2020/2021, Mathematics Education study program, FKIP USN Kolaka. Based on the results of descriptive statistical calculations, the average value of the spread of probabilistic thinking instruments was 77.9583. This means that some students get scores above 77.9583 and some below 77.9583. Variance is the variance of the data obtained from the multiple of the standard deviation. The variance value for the probabilistic thinking variable is 81,607. This means that in working on probabilistic thinking questions it has a very high level of diversity. Std Deviation is a measure of the spread of data from its average value. In this case the std deviation of the probabilistic thinking variable is 9.03365. This means that in terms of critical thinking, the distribution of data from the average score is in the good category. The full results can be seen in the following table

Table 1. Thinking test result Data Probabilistic

Statistics		
Probabilistic thinking test		
N	Valid	24
	missing	0
Means		77.9583
std. Error of Means		1.84399
Median		80,0000
Mode		80.00
std. Deviation		9.03365
Variances		81607
Range		35.00
Minimum		60.00
Maximum		95.00
sum		1871.00

For problem solving test results obtained an average value of 82.1250. This means that some students in answering the questions scored above 82.1250 and some were below 82.1250. Variance is the variance of the data obtained from the multiple of the standard deviation. The variance value for the ability to solve mathematical problems is 60,462. This means that in working on critical thinking questions it has a very high level of diversity. Std Deviation, which is a measure of the spread of data from the average value. In this case the std deviation of the ability to solve math problems is 7.77573. This means that in the mathematical problem solving instrument the distribution of data from the average value is included in the good category. The full results can be seen in the following table.

Table 2. Data on the results of the Binomial Distribution Problem Solving Test

Statistics		
Binomial distribution problem solving test		
N	24	24
	0	0
Means		82.1250
std. Error of Means		1.58721
Median		80.5000
Mode		75.00
std. Deviation		7.77573
Variances		60,462
Range		26.00
Minimum		70.00
Maximum		96.00
sum		1971.00

After doing the description on the data that has been obtained, The next step is to test the analysis prerequisites, namely the normality test and linearity test. Normality test done to find out whether the data obtained is located at normal level or not. Besides Therefore, a normality test was performed for determine the step in the test statistics used for answer the research hypothesis whether parametric statistics or non-parametric. In the normality test, the authors use a program on the computer, namely SPSS 20.0. The following is the presentation of normality test data:

Table 3. Data Normality Test

Tests of Normality ^a						
	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
	Statistics	df	Sig.	Statistics	df	Sig.
Unstandardized Residuals	.146	24	.199	.955	24	.349

The data states that the normality test is based on value the probability compared to the value of degrees of freedom is equal to $\alpha = 0.05$. After being tested using the Kolmogorov-Smirnov test as shown in table 3, the value of the normality test is 0.199. This value is declared significant because the probability/significance value is more than 0.05 ($0.199 > 0.05$). Thus, the probabilistic thinking data and problem solving on the binomial distribution are normally distributed.

The linearity test is one of the prerequisites for conducting a correlation or linear regression analysis test, which aims to determine whether each variable has a significant relationship or not. The SPSS 20.0 computer program is used to simplify the linearity test. The data from the linearity test results are as follows:

Table 4. ANOVA Table

ANOVA Table							
			Sum of Squares	df	Mean Square	F	Sig.
Y * X	Betwen Groups	(Combine)	777,758	13	59,828	.976	.526
		Linearity	359,223	1	359.23	5,861	.036
		Deviation from Linearity	418,536	12	34,878	.569	.824
	Within Groups		612,867	10	61,287		
	Total		1390.63	23			

the results of the linearity test it is known that the significance value of the Deviation from Linearity is 0.824. Because the significance is more than $\alpha = 0.05$, it can be concluded that there is a linear relationship between probabilistic thinking variables and problem solving variables in the binomial distribution.

Furthermore, the Regression Equation, can be determined using SPSS program version 20.0 which is presented in the following table:

Table 5. Simple Linear Regression

Coefficients ^a						
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	std. Error	Betas		
1	(Constant)	48020	12,400		3,873	001
	X	.437	.158	.508	2,768	011

Based on the calculation results in the Coefficients table in above, then the resulting model equation is: $\hat{Y} = 48.020 + 0.437X$. A constant of 48,020 states that if the critical thinking test scores are 0, 0, 437 then the problem solving in the student binomial distribution is 48,020. The

regression coefficient of 0.437 states that each additional value of 1 on the probabilistic thinking test will increase problem solving in the student binomial distribution of 0.437.

Model goodness-of-fit test was conducted to determine the influence between the independent variables and the dependent variable. To find out how big the percentage of students' probabilistic thinking influences problem solving in the binomial distribution, a coefficient of determination test is carried out as follows:

Table 6. Model Goodness Test

Summary Model ^b				
Model	R	R Square	Adjusted R Square	std. Error of the Estimate
1	.508 ^a	.258	.225	6.84704
a. Predictors: (Constant), X				
b. Dependent Variable: Y				

It is known that the value of $R = 0.508$ and $R^2 = 0.258$. This means that probabilistic thinking contributes to solving problems in the binomial distribution of 50.8% and the remaining 25.8% is determined by other factors.

Hypothesis testing was conducted to find out whether the independent variable (X) has a significant effect on the dependent variable (Y). Significant means that the effect that occurs can apply to the population (generalized). Data from simple linear regression analysis are presented in the following table:

Table 7. Simple Linear Regression

ANOVA ^a						
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	359,223	1	359,223	7,662	.011
	residual	1031,402	22	46,882		
	Total	1390625	23			
a. Dependent Variables: y						
b. Predictors: (Constant), x						

From the results of the Simple Linear Regression test it is known that the significance value of the Regression is 0.011. Because the significance is less than $\alpha = 0.05$, it can be concluded that students think probabilistically has a significant influence on solving problems in the binomial distribution.

Discussion In this study, researchers tested a population group that took a probabilistic thinking test. Based on the research that has been done, the results obtained state that probabilistic thinking affects students' ability to solve binomial distribution problems, this is in line with research conducted by (Rott et al., 2021; Yousif & Abdellahi, 2013) that in solving probability problems both sample space, disjoint events, partitions, conditional probabilities, Bayes theorem and opportunity distribution, the role of probabilistic thinking is very influential in increasing one's learning because with the concept of probability for an event, one can know various fields such as social phenomena and others. Before probabilistic thinking tests and

binomial distribution problem solving were distributed, the authors conducted validity tests, reliability tests, index of difficulty and discriminatory power. The point is to find out whether these questions can be used as a research instrument.

The results of the data analysis of the two tests are the probabilistic thinking test and solving test binomial distribution problem obtained an average of 77.9583 for probabilistic thinking tests and 82.1250 for the test solving the binomial distribution problem. The next step, the researcher conducted a requirements analysis test, namely the normality test and linearity test with the help of SPSS 20.0 software. The results of the two tests state that the data is normally distributed and has a linear relationship. The normality test results obtained a value of 0.349. This value was declared significant because the probability/significance value was more than 0.05 ($0.349 > 0.05$). Thus, the two data are distributed normal. The results of the calculation of the linearity test show that $r = 0.824$ is more than $\alpha = 0.05$, so probabilistic thinking and solving binomial distribution problems have a linear relationship. Because the data is normally distributed and has a linear relationship, the researchers carried out the next steps, namely the regression equation, testing the goodness of the model and testing the hypothesis. From the discussion of research on the effect of probabilistic thinking on undergraduate students' ability to solve the binomial distribution problem above, it can be understood that theoretically probabilistic thinking can improve the ability to solve binomial distribution problems in undergraduate students of mathematics education at USN Kolaka Class 2020/2021 .

This is in line with the results of research by (Di Paola et al., 2018; Teigen & Keren, 2020) that in solving probability distribution problems, both discrete distributions and continuous distributions, of course requires a random experiment to be able to find out how big the chance is in solving the problem binomial distribution, therefore. Every time you determine a result, be it success or failure, you must first do an experiment.

CONCLUSION

Based on data analysis of probabilistic thinking variables and solution to problem on the binomial distribution described in Chapter IV, it can be concluded that: 1) Undergraduate students in solving probabilistic thinking questions as a whole based on table 1 are categorized as good. This can be seen by the average student score of 77.9583 ; 2) The ability of undergraduate students in solving problems on the binomial distribution obtained from the post-test results given to students as a whole based on table 2 categorized as good. This can be seen with an average score of 82.1250. Based on Table 8 obtained F value = 7.662 with sig = $0.011 < \alpha = 0.05$, thus H_0 is rejected and H_a is accepted. Proved that think probabilistic significant effect to solving the problem of the binomial distribution. From the results data analysis obtained price r of 0.5 08 according to table 7 which fall into the sufficient criteria. While the contribution of thinking probabilistic to ability undergraduate students in solving binomial distribution problems is equal to 25.80 % and the rest of 74.20 % is determined by another factor. As for the similarities variable Y regression on variable X is: $\hat{Y} = 48,020 + 0,437X$. A constant of 48, 020 states that if the value probabilistic thinking is 0.437 then the undergraduate student's ability to solve the problem of the binomial distribution is 48,020. The regression coefficient is 0.437 stated that each additional value of 1 in probabilistic thinking would increase undergraduate students' ability to solve binomial distribution problems by 0.437.

Based on the results of the research and discussion as well as the conclusions stated above, the researcher provides the following suggestions: 1) As a follow-up to the research results, the following are put forward some suggestions which are expected to be contribute thoughts in upgrading undergraduate students' abilities in solve the problem binomial distribution; 2) In the learning process of mathematical statistics I in the lecture room, undergraduate students better think more probabilistic to increase students' abilities in solve the problem binomial distribution; 3) In the implementation of the test on the learning process, should be a lecturer consider questions probabilistic thinking for Upgrade students' abilities in solve the problem binomial distribution; 4) college should recommend test probabilistic thinking as use evaluation for Upgrade students' abilities in solve the problem distribution binomial.

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